Chapter 4
ENERGY ANALYSIS OF CLOSED SYSTEMS

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Objectives

- Examine the moving boundary work or $P\,dV$ work commonly encountered in reciprocating devices such as automotive engines and compressors.
- Identify the first law of thermodynamics as simply a statement of the conservation of energy principle for closed (fixed mass) systems.
- Develop the general energy balance applied to closed systems.
- Define the specific heat at constant volume and the specific heat at constant pressure.
- Relate the specific heats to the calculation of the changes in internal energy and enthalpy of ideal gases.
- Describe incompressible substances and determine the changes in their internal energy and enthalpy.
- Solve energy balance problems for closed (fixed mass) systems that involve heat and work interactions for general pure substances, ideal gases, and incompressible substances.
Moving boundary work ($P \, dV$ work): The expansion and compression work in a piston-cylinder device.

$$\delta W_b = F \, ds = PA \, ds = P \, d\mathcal{V}$$

$$W_b = \int_1^2 P \, d\mathcal{V} \quad (kJ)$$

Quasi-equilibrium process: A process during which the system remains nearly in equilibrium at all times.

$W_b$ is positive $\rightarrow$ for expansion

$W_b$ is negative $\rightarrow$ for compression

A gas does a differential amount of work $\delta W_b$ as it forces the piston to move by a differential amount $ds$. The work associated with a moving boundary is called *boundary work*. The moving boundary
The area under the process curve on a $P$-$V$ diagram represents the boundary work.

\[
\text{Area} = \Lambda = \int_{1}^{2} dA = \int_{1}^{2} P \, dV
\]

The boundary work done during a process depends on the path followed as well as the end states.

The net work done during a cycle is the difference between the work done by the system and the work done on the system.
Polytropic, Isothermal, and Isobaric processes

Polytropic process: $P = C V^{-n}$  

Polytropic process

$W_b = \int_{1}^{2} P dV = \int_{1}^{2} C V^{-n} dV = C \frac{V_{2}^{-n+1} - V_{1}^{-n+1}}{-n + 1} = \frac{P_{2} V_{2} - P_{1} V_{1}}{1 - n}$

Polytropic and for ideal gas

$W_b = \frac{m R (T_2 - T_1)}{1 - n}$

When $n = 1$  
(isothermal process)

$W_b = \int_{1}^{2} P dV = \int_{1}^{2} C V^{-1} dV = P V \ln \left( \frac{V_{2}}{V_{1}} \right)$

Constant pressure process

$W_b = \int_{1}^{2} P dV = P_0 \int_{1}^{2} dV = P_0 (V_{2} - V_{1})$

What is the boundary work for a constant-volume process?

Schematic and $P-V$ diagram for a polytropic process.

$P V^n = C = \text{const.}$

$P_1 V_1^n = P_2 V_2^n$
ENERGY BALANCE FOR CLOSED SYSTEMS

Energy balance for any system undergoing any process

\[ \frac{E_{\text{in}} - E_{\text{out}}}{\text{kJ}} = \Delta E_{\text{system}} \]

Net energy transfer by heat, work, and mass
Change in internal, kinetic, potential, etc., energies

Energy balance in the rate form

\[ \frac{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}{\text{kW}} = \frac{dE_{\text{system}}/dt}{\text{kJ}} \]

Rate of net energy transfer by heat, work, and mass
Rate of change in internal, kinetic, potential, etc., energies

The total quantities are related to the quantities per unit time is

\[ Q = \dot{Q} \Delta t, \quad W = \dot{W} \Delta t, \quad \text{and} \quad \Delta E = (dE/dt)\Delta t \quad \text{(kJ)} \]

Energy balance per unit mass basis

\[ e_{\text{in}} - e_{\text{out}} = \Delta e_{\text{system}} \quad \text{(kJ/kg)} \]

Energy balance in differential form

\[ \delta E_{\text{in}} - \delta E_{\text{out}} = dE_{\text{system}} \quad \text{or} \quad \delta e_{\text{in}} - \delta e_{\text{out}} = de_{\text{system}} \]

Energy balance for a cycle

\[ W_{\text{net, out}} = Q_{\text{net, in}} \quad \text{or} \quad \dot{W}_{\text{net, out}} = \dot{Q}_{\text{net, in}} \]
Energy balance when sign convention is used (i.e., heat input and work output are positive; heat output and work input are negative).

For a cycle $\Delta E = 0$, thus $Q = W$.

The first law cannot be proven mathematically, but no process in nature is known to have violated the first law, and this should be taken as sufficient proof.
Energy balance for a constant-pressure expansion or compression process

General analysis for a closed system undergoing a quasi-equilibrium constant-pressure process. $Q$ is to the system and $W$ is from the system.

$$\frac{E_{\text{in}} - E_{\text{out}}}{\text{Net energy transfer by heat, work, and mass}} = \Delta E_{\text{system}}$$

Change in internal, kinetic, potential, etc., energies

$$Q - W = \Delta U + \Delta K_E + \Delta P_E$$

$$Q - W_{\text{other}} - W_b = U_2 - U_1$$

For a constant-pressure expansion or compression process:

$$\Delta U + W_b = \Delta H$$

An example of constant-pressure process

$$W_{e,\text{in}} - Q_{\text{out}} - W_b = \Delta U$$

$$W_{e,\text{in}} - Q_{\text{out}} = \Delta H = m(h_2 - h_1)$$

$Q = W_{\text{other}} - P_0(V_2 - V_1) = U_2 - U_1$

$Q - W_{\text{other}} = (U_2 + P_2V_2) - (U_1 + P_1V_1)$

$H = U + PV$

$Q - W_{\text{other}} = H_2 - H_1$
SPECIFIC HEATS

Specific heat at constant volume, $c_v$: The energy required to raise the temperature of the unit mass of a substance by one degree as the volume is maintained constant.

Specific heat at constant pressure, $c_p$: The energy required to raise the temperature of the unit mass of a substance by one degree as the pressure is maintained constant.

$\begin{align*}
\text{Constant-volume and constant-pressure specific heats } c_v \text{ and } c_p \\
\text{(values are for helium gas).}
\end{align*}$
• The equations in the figure are valid for *any* substance undergoing *any* process.
• \( c_v \) and \( c_p \) are properties.
• \( c_v \) is related to the changes in *internal energy* and \( c_p \) to the changes in *enthalpy*.
• A common unit for specific heats is kJ/kg \( \cdot \) °C or kJ/kg \( \cdot \) K. **Are these units identical?**

The specific heat of a substance changes with temperature.

**True or False?**
\( c_p \) is always greater than \( c_v \).
Joule showed using this experimental apparatus that \( u = u(T) \).

For ideal gases, \( u, h, c_v, \) and \( c_p \) vary with temperature only.

Internal energy and enthalpy change of an ideal gas:

\[
\begin{align*}
\Delta u &= u_2 - u_1 = \int_1^2 c_v(T) \, dT \\
\Delta h &= h_2 - h_1 = \int_1^2 c_p(T) \, dT
\end{align*}
\]
• At low pressures, all real gases approach ideal-gas behavior, and therefore their specific heats depend on temperature only.
• The specific heats of real gases at low pressures are called *ideal-gas specific heats*, or *zero-pressure specific heats*, and are often denoted $c_{p0}$ and $c_{v0}$.

- $u$ and $h$ data for a number of gases have been tabulated.
- These tables are obtained by choosing an arbitrary reference point and performing the integrations by treating state 1 as the reference state.

<table>
<thead>
<tr>
<th>AIR</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$, K</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>300</td>
</tr>
<tr>
<td>310</td>
</tr>
</tbody>
</table>

In the preparation of ideal-gas tables, 0 K is chosen as the reference temperature.
Internal energy and enthalpy change when specific heat is taken constant at an average value

\[
\begin{align*}
    u_2 - u_1 &= c_{v,\text{avg}}(T_2 - T_1) \\
    h_2 - h_1 &= c_{p,\text{avg}}(T_2 - T_1)
\end{align*}
\]

(kJ/kg)

For small temperature intervals, the specific heats may be assumed to vary linearly with temperature.

The relation \( \Delta u = c_v \Delta T \) is valid for any kind of process, constant-volume or not.
Three ways of calculating $\Delta u$ and $\Delta h$

1. By using the tabulated $u$ and $h$ data. This is the easiest and **most accurate** way when tables are readily available.

2. By using the $c_v$ or $c_p$ relations (Table A-2c) as a function of temperature and performing the integrations. This is very inconvenient for hand calculations but quite desirable for computerized calculations. The results obtained are **very accurate**.

3. By using average specific heats. This is very simple and certainly very convenient when property tables are not available. The results obtained are **reasonably accurate** if the temperature interval is not very large.

Three ways of calculating $\Delta u$. 

- $\Delta u = u_2 - u_1$ (table)
- $\Delta u = \int_{1}^{2} c_v(T) \, dT$
- $\Delta u \equiv c_{v,\text{avg}} \Delta T$
Specific Heat Relations of Ideal Gases

\[ h = u + RT \]
\[ dh = du + R \, dT \]
\[ dh = c_p \, dT \quad \text{and} \quad du = c_v \, dT \]

The relationship between \( c_p, c_v \) and \( R \)

\[ c_p = c_v + R \quad (\text{kJ/kg} \cdot \text{K}) \]

On a molar basis

\[ \overline{c}_p = \overline{c}_v + R_u \quad (\text{kJ/kmol} \cdot \text{K}) \]

\[ k = \frac{c_p}{c_v} \]

Specific heat ratio

- The specific ratio varies with temperature, but this variation is very mild.
- For monatomic gases (helium, argon, etc.), its value is essentially constant at 1.667.
- Many diatomic gases, including air, have a specific heat ratio of about 1.4 at room temperature.

AIR at 300 K

\[
\begin{align*}
c_v &= 0.718 \text{ kJ/kg} \cdot \text{K} \\
R &= 0.287 \text{ kJ/kg} \cdot \text{K} \\
\end{align*}
\]

\[
\begin{align*}
c_p &= 1.005 \text{ kJ/kg} \cdot \text{K} \\
\end{align*}
\]

or

\[
\begin{align*}
\overline{c}_v &= 20.80 \text{ kJ/kmol} \cdot \text{K} \\
R_u &= 8.314 \text{ kJ/kmol} \cdot \text{K} \\
\end{align*}
\]

\[
\begin{align*}
\overline{c}_p &= 29.114 \text{ kJ/kmol} \cdot \text{K} \\
\end{align*}
\]

The \( c_p \) of an ideal gas can be determined from a knowledge of \( c_v \) and \( R \).
**INTERNAL ENERGY, ENTHALPY, AND SPECIFIC HEATS OF SOLIDS AND LIQUIDS**

**Incompressible substance**: A substance whose specific volume (or density) is constant. Solids and liquids are incompressible substances.

The specific volumes of incompressible substances remain constant during a process.

\[
\begin{align*}
\text{LIQUID} & : v_l = \text{constant} \\
\text{SOLID} & : v_s = \text{constant}
\end{align*}
\]

The \(c_v\) and \(c_p\) values of incompressible substances are identical and are denoted by \(c\).

IRON

\[
\begin{align*}
25^\circ\text{C} & \\
= 0.45 \text{ kJ/kg} \cdot ^\circ\text{C}
\end{align*}
\]
Internal Energy Changes

\[ du = c_v \, dT = c(T) \, dT \]

\[ \Delta u = u_2 - u_1 = \int_1^2 c(T) \, dT \quad (\text{kJ/kg}) \]

\[ \Delta u \approx c_{\text{avg}} (T_2 - T_1) \quad (\text{kJ/kg}) \]

Enthalpy Changes

\[ h = u + P \nu \]

\[ dh = du + \nu \, dP + P \, d\nu = du + \nu \, dP \]

\[ \Delta h = \Delta u + \nu \, \Delta P \approx c_{\text{avg}} \Delta T + \nu \, \Delta P \quad (\text{kJ/kg}) \]

For solids, the term \( \nu \, \Delta P \) is insignificant and thus \( \Delta h = \Delta u \approx c_{\text{avg}} \Delta T \). For liquids, two special cases are commonly encountered:

1. Constant-pressure processes, as in heaters (\( \Delta P = 0 \)): \( \Delta h = \Delta u \approx c_{\text{avg}} \Delta T \)

2. Constant-temperature processes, as in pumps (\( \Delta T = 0 \)): \( \Delta h = \nu \, \Delta P \)

\[ h_{@P,T} \approx h_{f@T} + \nu_{f@T}(P - P_{\text{sat} @ T}) \]

A more accurate relation than \( h_{@P,T} \approx h_{f@T} \)

The enthalpy of a compressed liquid
Summary

• Moving boundary work
  ✓ $W_b$ for an isothermal process
  ✓ $W_b$ for a constant-pressure process
  ✓ $W_b$ for a polytropic process

• Energy balance for closed systems
  ✓ Energy balance for a constant-pressure expansion or compression process

• Specific heats
  ✓ Constant-pressure specific heat, $c_p$
  ✓ Constant-volume specific heat, $c_v$

• Internal energy, enthalpy, and specific heats of ideal gases
  ✓ Specific heat relations of ideal gases

• Internal energy, enthalpy, and specific heats of incompressible substances (solids and liquids)