Introduction

The reciprocating pump is a positive displacement pump as it sucks and raises the liquid by actually displacing it with a piston/plunger that executes a reciprocating motion in a closely fitting cylinder. The amount of liquid pumped is equal to the volume displaced by the piston.

The pumps designed with disk pistons create pressures up to 25 bar and the plunger pumps built up still higher pressures. Discharge from these pumps is almost wholly dependent on the pump speed.

The total efficiency of a reciprocating pump is about 10 to 20% higher than a comparable centrifugal pump.

Reciprocating pumps for industrial uses have almost become obsolete owing to their high capital cost as well as maintenance cost as compared to that of centrifugal pumps. However, small hand-operated pumps such as cycle pumps, football pumps, kerosene pumps, village well pumps and pumps used as important parts of hydraulic jack etc. still find wide applications. The reciprocating pump is best suited for relatively small capacities and high heads. This type of pump is very common in oil drilling operations.

The reciprocating pump is generally employed for:

(i) Light oil pumping,
(ii) Feeding small boilers condensate return, and
(iii) Pneumatic pressure systems.

Classification of Reciprocating Pumps

Reciprocating pumps are classified as follows:

1. According to the water being in contact with piston:
   (i) Single-acting pump ...water is in contact with one side of the piston
   (ii) Double-acting pump ...water is in contact with both sides of the piston.

2. According to number of cylinders:
   * (i) Single cylinder pump
   (ii) Double cylinder pump (or two throw pump)

   (iii) Triple cylinder pump (or three throw pump)
   (iv) Duplex double-acting pump (or four throw pump)
   (v) Quintuplex pump or (five throw pump).

In general the reciprocating pumps having more than one cylinder are known as multi-cylinder pumps.
Main components of single acting reciprocating pumps

Fig. 1  Schematic view of single acting reciprocating pump.

Components of double acting reciprocating pumps

Fig. 2  Double-acting reciprocating pump.
Discharge, Work done and Power Required to Drive Reciprocating Pump

Single-acting reciprocating pump

Consider a single-acting reciprocating pump:

Let, \( D = \) diameter of the cylinder, \( \text{m} \)

\[ A = \text{cross-sectional area of the piston/cylinder} = \frac{\pi}{4} D^2 \text{ m}^2 \]

\( r = \) radius of crank, \( \text{m} \)

\( N = \) speed of the crank, r.p.m.

\( L = \) length of the stroke (\( = 2r \)), \( \text{m} \)

\( h_s = \) height of the centre of the cylinder above the liquid surface, \( \text{m} \) and

\( h_d = \) height to which the liquid is raised above the centre of the cylinder, \( \text{m} \)

Volume of liquid sucked in during suction stroke = \( A \times L \)
\[ Q = A \times L \times \frac{N}{60} \]

Weight of water delivered per second, \[ W = wQ = \frac{wALN}{60} \]

Work done per second = weight of water lifted/sec \times total height through which liquid is lifted

\[ = W(h_s + h_d) = \frac{wALN}{60}(h_s + h_d) \]

\[ \therefore \text{Power required to drive the pump} = \frac{wALN}{60 \times 1000}(h_s + h_d) \text{ kW} \]

(where \( w \) = weight density of liquid in N/m\(^3\))

**Double-acting reciprocating pump**

Let

\[ D = \text{diameter of the piston,} \]
\[ d = \text{diameter of the piston rod,} \]
\[ A_{pr} = \text{cross-sectional area of the piston rod} = \frac{\pi}{4}d^2 \]

Area on one side of the piston, \( A = \frac{\pi}{4}D^2 \)

Area on other side of the piston where piston rod is connected to the piston,

\[ A' = A - A_{pr} = \frac{\pi}{4}D^2 - \frac{\pi}{4}d^2 = \frac{\pi}{4}(D^2 - d^2). \]

Volume of liquid delivered in one revolution of crank

\[ = AL + A'L = (A + A')L = \left[ \frac{\pi}{4}D^2 + \frac{\pi}{4}(D^2 - d^2) \right]L \]

\[ \therefore \text{Discharge of the pump per second} = \left[ \frac{\pi}{4}D^2 + \frac{\pi}{4}(D^2 - d^2) \right]L \times \frac{N}{60} \]

If the diameter of the piston rod ‘d’ is very small as compared to the diameter of the piston ‘D’ then it can be neglected and hence discharge of the pump per second will become

\[ Q = \left( \frac{\pi}{4}D^2 + \frac{\pi}{4}D^2 \right) \times \frac{LN}{60} = 2 \times \frac{\pi}{4}D^2 \times \frac{LN}{60} = \frac{2ALN}{60} \]

Evidently the output of a double-acting pump is two-times that of a single acting pump.

**Work done per second = weight of water delivered \times total height through which liquid is lifted**

\[ = \left( w \times \frac{2ALN}{60} \right) \times (h_s + h_d) \]

\[ = \frac{2wALN}{60}(h_s + h_d) \]

Power required to drive the pump, \[ P = \frac{2wALN}{60 \times 1000}(h_s + h_d) \text{ kW} \]

(where \( w \) = weight density of liquid in N/m\(^3\))

**Coefficient of Discharge and Slip of Reciprocating Pump**

**Coefficient of discharge**

In a reciprocating pump, the actual discharge \( (Q_{act}) \) is always slightly different from the theoretical discharge \( (Q_{th}) \) due to following reasons:
(i) Leakage through the valves, glands and piston packing,
(ii) Imperfect operation of the valves (suction and discharge), and
(iii) Partial filling of cylinder by the liquid.

The ratio between actual discharge and theoretical discharge is known as the co-efficient of discharge (C) of the pump. That is

\[ C_d = \frac{\text{actual discharge}}{\text{theoretical discharge}} = \frac{Q_{act.}}{Q_{th.}} \]

When the value of \( C_d \) is expressed in percentage, it is known as 'volumetric efficiency'.

The difference between the theoretical discharge and actual discharge is called the 'slip' of the pump. That is

\[ \text{Slip} = Q_{th.} - Q_{act.} \]

But the slip is often expressed in percentage which is given by,

\[ \% \text{ Slip} = \frac{Q_{th.} - Q_{act.}}{Q_{th.}} \times 100 = \left(1 - \frac{Q_{act.}}{Q_{th.}}\right) \times 100 = (1 - C_d) \times 100 \]

The percentage of slip for the pumps maintained in good condition is of the order of 2% or even less.

**Negative slip.** In most of the reciprocating pumps \( Q_{act.} \) is less than \( Q_{th.} \); in such a case the value of \( C_d \) is less than unity and the slip of the pump is 'positive'. However, in some cases \( Q_{act.} \) may be more than \( Q_{th.} \); in such a case \( C_d \) is more than unity and the slip will be 'negative'. The slip will be negative when there is a direct connection between the suction and delivery sides before the end of suction stroke. This happens if the momentum of liquid in the suction pipe is large enough to open the delivery valve before the beginning of delivery stroke. The negative slip is possible in case of pumps having long suction pipe and a short delivery pipe, especially when these are operating at high speeds.

**Effect of Acceleration of Piston on Velocity and Pressure in the Suction and Delivery Pipes**

If the crank rotates uniformly and the length of connection rod is enough compared to the radius of crank, the piston makes simple harmonic. This causes acceleration during the first half of the stroke and deceleration during the second half of the stroke.
Let, \( A \) = area of the cylinder, \\
a = area of the pipe (suction or delivery), \\
l = length of pipe (suction or delivery), \\
r = radius of the crank, and \\
\omega = \text{angular speed of the crank in rad/s.}

The crank is rotating with an angular velocity \( \omega \) and let in time \( t \) seconds, the crank turns through angle \( \theta \) (in radians) from I.D.C. (inner dead centre). The displacement of the piston in time \( t \) is \( x \)

Now, angle turned by the crank in time \( t \), \( \theta = \omega t = \frac{2\pi N}{60} \times t \) 

(where \( N \) = rotational speed of crank in r.p.m.)

The corresponding distance (\( x \)) travelled by the piston, 
\[ x = r - r \cos \theta = r (1 - \cos \theta) = r (1 - \cos \omega t) \]

Velocity of the piston, 
\[ V = \frac{dx}{dt} = \frac{d}{dt} [r (1 - \cos \omega t)] = \frac{d}{dt} (r - r \cos \omega t) \]

or
\[ V = r \omega \sin \omega t \]

Acceleration of the piston, 
\[ a_p = \frac{dV}{dt} = \frac{d}{dt} (r \omega \sin \omega t) = \omega^2 r \cos \omega t \]

Now from continuity considerations, the volume of liquid flowing from the pipe equals the volume of liquid flowing into the cylinder.

\[ \therefore \] Velocity of liquid in the pipe (\( v \)) \times area of pipe (\( a \)) = velocity of piston (\( V \)) \times area of cylinder (\( A \))

\[ v = \frac{AV}{a} = A \frac{a}{a} \omega \sin \omega t \]

Acceleration of liquid in pipe = \( \frac{d}{dt} (v) = \frac{d}{dt} \left[ \frac{A}{a} \omega \sin \omega t \right] \]

\[ = \frac{A}{a} \omega^2 r \cos \omega t \]

Mass of water in pipe = density \times volume of liquid in pipe = \( pA \)

Force required to accelerate the water in the pipe = mass \times acceleration

\[ = pA \frac{A}{a} \omega^2 r \cos \omega t \]

\[ \therefore \] Intensity of pressure due to acceleration

\[ = \frac{\text{force required to accelerate the liquid}}{\text{area of pipe}} = \frac{pA}{a} \omega^2 r \cos \omega t \]

\[ = p\left( \frac{A}{a} \right) \omega^2 r \cos \theta \]

\[ \therefore \] Pressure head due to acceleration

\[ h_a = \frac{\text{intensity of pressure}}{\text{weight density of liquid (\( w \))}} = \frac{pA}{ag} \omega^2 r \cos \theta \]

\[ = \frac{l}{g} \frac{A}{a} \omega^2 r \cos \theta \quad (\because \ w = \rho g) \]

The pressure head due to acceleration in the suction and delivery pipes is obtained by
using subscripts 's' and 'd' respectively

\[ h_s = \frac{1}{2} \times \frac{A}{a} \omega^2 r \cos \theta \]

\[ h_d = \frac{1}{2} \times \frac{A}{a_d} \omega^2 r \cos \theta \]

Note: It may be noted that for any stroke, the angular displacement \( \theta \) is measured from the instant of commencement of that stroke. In case of suction stroke the piston moves outward and \( \theta \) is measured from I.D.C. (inner dead centre) and during delivery stroke the piston moves inward and \( \theta \) is measured from O.D.C (outer dead centre).

The values of '\( h_a \)' for different values of \( \theta \) are:

(i) When \( \theta = 0^\circ \) (i.e. the beginning of the stroke), \( h_a = \frac{1}{2} \frac{A}{a} \omega^2 r \quad (\because \cos 0^\circ = 1) \)

(ii) When \( \theta = 90^\circ \) (i.e. the middle of the stroke), \( h_a = 0 \quad (\because \cos 90^\circ = 0) \)

(iii) When \( \theta = 180^\circ \) (i.e. the end of the stroke), \( h_a = -\frac{1}{2} \frac{A}{a} \omega^2 r \quad (\because \cos 180^\circ = -1) \)

\[ h_a \text{ maximum, } (h_a)_{\text{max}} = \frac{1}{2} \frac{A}{a} \omega^2 r \]

For \( 0^\circ < \theta < 90^\circ \), \( h_a \) has +ve values and for \( 90^\circ < \theta < 180^\circ \), \( h_a \) has -ve values, thereby indicating that for the first half of the stroke there is acceleration head development and in the later half of the stroke retardation head is developed.

In case the connecting rod is not very long as compared to crank length then it cannot be assumed that the piston has a simple harmonic motion and in that case the pressure head, \( h_a \) is given by,

\[ h_a = \frac{1}{2} \frac{A}{a} \omega^2 r \cos \theta \left( \cos \theta + \frac{\cos 2\theta}{n} \right) \]

where \( n = \) ratio of the length of connecting rod to the crank length.

we have

(i) When \( \theta = 0^\circ \) (i.e. the beginning of the stroke), \( h_a = \frac{1}{2} \frac{A}{a} \omega^2 r \left( 1 + \frac{1}{n} \right) \)

(ii) When \( \theta = 90^\circ \) (i.e. the middle of stroke), \( h_a = 0 \)

(iii) When \( \theta = 180^\circ \) (i.e. at the end of the stroke), \( h_a = \frac{1}{2} \frac{A}{a} \omega^2 r \left( 1 - \frac{1}{n} \right) \)

(a) Effect of variation of velocity on friction in pipes:

The liquid flowing through suction and delivery pipes causes loss of head due to friction, which is given by Darcy-Weisbach equation as:

\[ h_f = \frac{f l v^2}{d \times 2g} \]

where, \( f = \) co-efficient of friction,

\( l = \) length of the pipe,

\( d = \) diameter of the pipe, and
\( v = \text{velocity of liquid in the pipe.} \)

Also the velocity of liquid in the pipe, \( v = \frac{A}{a} \omega r \sin \theta = \frac{A}{a} \omega r \sin \theta \)

Substituting the value of \( \nu \) in (i), we get

\[
\frac{h_f}{d \times 2g} = \left( \frac{A}{a} \omega r \sin \theta \right)^2
\]

The variation of \( h_f \) with \( \theta \) is parabolic. The values of \( h_f \) for suction and delivery pipes are obtained from eqn. above by using subscripts 's' for suction pipe and 'd' for delivery pipe as:

\[
h_f = \frac{f_1}{d_s \times 2g} \left( \frac{A}{a_s} \omega r \sin \theta \right)^2
\]

\[
h_f = \frac{f_1}{d_d \times 2g} \left( \frac{A}{a_d} \omega r \sin \theta \right)^2
\]

The loss of head due to friction \( (h_f) \) in pipes given by eqn. above varies with \( \theta \) as:

(\( i \)) When \( \theta = 0^\circ \) (i.e. the beginning of the stroke), \( h_f = 0 \)

(\( ii \)) When \( \theta = 90^\circ \) (i.e. the middle of the stroke), \( h_f = \frac{f_1}{d \times 2g} \left( \frac{A}{a} \omega r \right)^2 \)

(\( iii \)) When \( \theta = 180^\circ \) (i.e. the end of the stroke), \( h_f = 0 \)

Maximum value of loss of head due to friction,

\[
(h_f)_{\text{max}} = \frac{f_1}{d \times 2g} \left( \frac{A}{a} \omega r \right)^2
\]

**Indicator Diagrams**

The indicator diagram of a reciprocating pump is the diagram which shows the pressure head of the liquid in the cylinder corresponding to any position during the suction and delivery strokes. It is a graph between pressure head and stroke length of the piston for one complete revolution (pressure head is taken as ordinate and stroke length as abscissa).

**Ideal indicator diagram**

The indicator diagram obtained by neglecting the loss of head due to friction in the suction and delivery pipes and the effect of acceleration of piston, is known as an ideal indicator diagram. Such diagram for a single-cylinder single-acting pump is shown in Fig. 5, the line EF represents the atmospheric pressure head \( H_{\text{atm}} = \frac{p_a}{\gamma} \) equal to 10.3 m of water.

Let, \( h_s = \text{suction head, and} \)

\( h_d = \text{delivery head.} \)

* The pressure head in the cylinder (represented by line AB) during suction stroke, is constant and equal to suction head \( (h_s) \) which is below the atmospheric pressure head \( H_{\text{atm}} \) by a height \( h_s \). The absolute pressure head in cylinder during the suction stroke will be \( (H_{\text{atm}} - h_s) \); it is shown by ordinate.

\[
\text{Length of stroke (L)}
\]

**Fig. 5 Ideal indicator diagram.**
AS at the beginning of the stroke and by ordinate BT at the end of stroke and, is uniform throughout the stroke.

- During the delivery stroke the pressure head in the cylinder (represented by line CD) is constant and equal to delivery head \( h_d \). The uniform absolute pressure head throughout the delivery stroke is \( h_d + h_{sim} \) and is denoted by the ordinate TC or SD.

The work done by the pump per second is

\[
\frac{w \times AN}{60} \times (h_s + h_d)
\]

\[
= K \times L \times (h_s + h_d)
\]

\[
\propto L \times (h_s + h_d)
\]

But from Fig. 20-7, the area of indicator diagram ABCDA

\[
= AB \times BC = AB (BF + FC) = L (h_s + h_d)
\]

From (i) and (ii), we have

Work done by the pump \( \propto \) area of indicator diagram

Thus, work done by the pump second = \( \frac{wAN}{60} \times \) area of indicator diagram

If the pump is double-acting, neglecting the area of the piston rod, work done per second is proportional to twice the area of the indicator diagram.

**Effect of acceleration in suction and delivery pipes on indicator diagram**

The effect of acceleration in suction and delivery pipes is discussed below:

(a) **Effect of acceleration in the suction pipe:**

As the piston (considering it as the beginning of the stroke) moves outward, it should create not only a negative pressure equal to the suction head \( h_s \) but it should also accelerate the liquid. If \( h_{as} \) is the acceleration head, then total negative pressure head at the beginning of the suction stroke is \( h_s + h_{as} \), the ordinate EA', absolute pressure head at this point is denoted by ordinate A'\( S \). So that separation does not take place, the absolute pressure at the beginning of stroke should not fall below the vapour pressure.

If \( l_s \) and \( a_s \) are length and cross-sectional area of the suction pipe respectively, then:

(i) **At the beginning of the suction stroke:**

The accelerating head, \( h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \omega^2 r \)

Negative pressure (vacuum) head, \( h_s + h_{as} = h_s + \frac{l_s}{g} \times \frac{A}{a_s} \omega^2 r \)

Absolute pressure head = \( H_{sim} - \left( h_s + \frac{l_s}{g} \times \frac{A}{a_s} \omega^2 r \right) \)

(ii) **At the middle of the suction stroke:**

The acceleration head, \( h_{as} = 0 \)
Negative pressure (vacuum) head = \( h_s \)

Absolute pressure head = \( H_{atm} - h_s \)

(iii) At the end of the suction stroke:

The acceleration head, \( h_{as} = -\frac{\frac{l_s}{g} \times \frac{A}{a_s} \omega^2 r}{1} \)

Negative pressure (vacuum) head = \( h_s + h_{as} = h_s - \frac{l_s}{g} \times \frac{A}{a_s} \omega^2 r \)

Absolute pressure head = \( H_{atm} - \left( h_s - \frac{l_s}{g} \times \frac{A}{a_s} \omega^2 r \right) \)

(b) Effect of acceleration in the delivery pipe:

In the beginning of delivery stroke the liquid in the delivery pipe is accelerated, while at the end of delivery stroke the liquid is retarded.

If \( l_d \) and \( a_d \) are the length and cross-sectional area of the delivery pipe respectively, then:

(i) At the beginning of delivery stroke:

Pressure head (gauge) = \( h_d + h_{ad} = h_d + \frac{l_d}{g} \times \frac{A}{a_d} \omega^2 r \)

(ii) At the middle of delivery stroke:

Pressure (gauge) head = \( h_d \) (\( \because h_{ad} = 0 \))

(iii) At the end of delivery stroke:

Pressure (gauge) head = \( h_d - \frac{l_d}{g} \times \frac{A}{a_d} \omega^2 r \)

Absolute pressure head = \( H_{atm} + h_d - \frac{l_d}{g} \times \frac{A}{a_d} \omega^2 r \)

The absolute pressure head (at the end of delivery stroke) given by above should not be less than vapour pressure to avoid separation.

It is evident from Fig. 6 that due to acceleration in suction and delivery pipes, the indicator diagram has changed from \( ABCD \) to \( A'B'C'D' \) but the area of indicator diagram remains unaltered. Thus the total work done remains the same. The main effect of the acceleration head is that it increases the negative head at the beginning of suction stroke. If the simple harmonic motion does not take place, the straight lines \( AB' \) and \( CD' \) will become slightly curved.